JOURNAL OF THE AMERICAN WATER RESOURCES ASSOCIATION

AMERICAN WATER RESOURCES ASSOCIATION

EMPIRICAL ESTIMATION OF STREAM DISCHARGE USING CHANNEL GEOMETRY IN LOW-GRADIENT, SAND-BED STREAMS OF THE SOUTHEASTERN PLAINS[1]

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ABSTRACT: Manning's equation is used widely to predict stream discharge (Q) from hydraulic variables when logistics constrain empirical measurements of in-bank flow events. Uncertainty in Manning's roughness (n_{M}) is the major source of error in natural channels, and sand-bed streams pose difficulties because flow resistance is affected by flow-dependent bed configuration. Our study was designed to develop and validate models for estimating Q from channel geometry easily derived from cross-sectional surveys and available GIS data. A database was compiled consisting of 484 Q measurements from 75 sand-bed streams in Alabama, Georgia, South Carolina, North Carolina (Southeastern Plains), and Florida (Southern Coastal Plain), with six New Zealand streams included to develop statistical models to predict Q from hydraulic variables. Model error characteristics were estimated with leave-one-site-out jackknifing. Independent data of 317 Q measurements from 55 Southeastern Plains streams indicated the model ($Q = A_{\mathrm{c}}{R_{\mathrm{H}}}^{0.6906}S^{0.1216}$; where A_{c} is the channel area, R_{H} is the hydraulic radius, and S is the bed slope) best predicted Q, based on Akaike's information criterion and root mean square error. Models also were developed from smaller Q range subsets to explore if subsets increased predictive ability, but error fit statistics suggested that these were not reasonable alternatives to the above equation. Thus, we recommend the above equation for predicting in-bank Q of unbraided, sandy streams of the Southeastern Plains.

(KEY TERMS: surface water hydrology; open-channel flow; rivers/streams; channel resistance; Manning's equation; hydraulics; discharge prediction; sand bed; Southeastern Plains.)

Sefick, Stephen A., Latif Kalin, Ely Kosnicki, Brad P. Schneid, Miller S. Jarrell, Chris J. Anderson, Michael H. Paller, and Jack W. Feminella, 2015. Empirical Estimation of Stream Discharge Using Channel Geometry in Low-Gradient, Sand-Bed Streams of the Southeastern Plains. *Journal of the American Water Resources Association* (JAWRA) 1-12. DOI: 10.1111/jawr.12278

INTRODUCTION

The Southeastern Plains are a large ecoregion of the United States (U.S.) spanning Maryland to Louisiana with generally low-gradient streams and sandy beds (Maloney *et al.*, 2005; Wilken *et al.*, 2011). Substrate in these lowland streams can be considered mobile because bed mobilization is initiated at flows as low as mean annual discharge (Copeland *et al.*, 2005). In contrast, high-gradient upland mountain streams contain predominately gravel and cobble

[1]Paper No. JAWRA-14-0086-P of the *Journal of the American Water Resources Association* (JAWRA). Received March 18, 2014; accepted December 1, 2014. **Discussions are open until six months from print publication**.

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beds, which require higher and less frequent bankfull discharges for mobilization (Doyle *et al.*, 2007). These contrasting stream types likely require strongly contrasting methods to estimate discharge empirically due to their large differences in bed mobility.

Logistical constraints often preclude empirical measurement of in-bank, high-flow events for developing stage-discharge relationships. The Chezy (1768 reported in Dingman (2009)) and Manning (1891) equations are frequently used to predict discharges from hydraulic variables. The Chezy equation is:

$$Q = CA_c R_H^{1/2} S_f^{1/2} \tag{1}$$

where Q is the discharge (m^3/s), A_c is the wetted channel area (m^2), R_H is the channel hydraulic radius (m), S_f is the energy slope (m/m), and C is the reach-specific bed resistance coefficient. Manning (1891) modified Chezy's equation resulting in an empirically preferable equation (Dingman and Sharma, 1997):

$$Q = (1/n_M)\, A_c R_H^{2/3} S_f^{1/2} \tag{2}$$

where n_M is the Manning's roughness coefficient (in SI units).

There is uncertainty in estimation of Manning's n_M, which may account for significant error in applying Manning's (1891) equation to natural channels (Dingman and Sharma, 1997; Bjerklie *et al.*, 2005; Lopez *et al.*, 2007). A major source of uncertainty in estimating n_M for natural channels is attributable to a nonrigid and highly dynamic bed (Yen, 1991), which is of particular concern in sand-bed streams because the bed can be mobilized as frequently as values approximating annual average Q (Copeland *et al.*, 2005). Three common methods for estimating n_M are: (1) selecting n_M from a table of typical values based on qualitative description of channel characteristics, (2) selecting n_M from photographs of channels displaying typical values, and (3) estimating n_M empirically using one of several equations relating hydraulic variables to n_M (Chow, 1959). Considerable work has been done relating n_M to instream variables in this context, with the Strickler (1923) equation being the most widely used:

$$n_M \cong 0.047 d_{50}^{1/6} \tag{3}$$

where d_{50} (m) is the 50th percentile of the bed particle size distribution (Ferguson, 2010). The Strickler equation provides an estimate of n_M that integrates bed particle size distribution (Strickler, 1923). The nonrigid bed of sand-bed streams also affects accuracy of estimating n_M with the Strickler equation (Yen, 1991).

Equation (3) above is appealing because it provides an empirical estimate of n_M, although n_M may be underestimated (Dingman, 2009; Ferguson, 2010). Sand-bed channels (generally characterized by $d_{50} \leq$ 2 mm) pose particular problems in n_M estimation because bed forms vary with Q, which can affect n_M (Simons and Richardson, 1966; Yen, 2002). Such variation makes it problematic to apply a single value of n_M to model Q in sand-bed streams, and this variability may exacerbate known difficulties in gauging Q accurately (Isaacson and Coonrod, 2011).

Brownlie (1983) developed equations for predicting velocity and depth of flow in sand-bed streams, which can be rearranged and substituted directly for Manning's equation to predict Q from hydraulic variables (Brownlie, 1983). These equations are an improvement over estimating n_M for use in Manning's equation, but they require empirical estimates of median and geometric standard deviation of bed particle size distributions. Furthermore, site-specific estimates of near-bed conditions and bed particle size distributions can be time- and/or cost-prohibitive, thus limiting the utility of Brownlie (1983) equations in regional assessments.

To overcome the above difficulties in estimating n_M, researchers have fit empirical regression models to hydraulic geometry and Q data without specific n_M estimation (Riggs, 1976; Bray, 1979; Dingman and Sharma, 1997; Bjerklie *et al.*, 2003, 2005; Lopez *et al.*, 2007). Dingman and Sharma (1997) and Bjerklie *et al.* (2003, 2005) fit statistical models to hydraulic variables to predict Q using a Manning-like model irrespective of bed particle composition, and accurately predicted Q at levels $>3\ m^3/s$. Lopez *et al.* (2007) conducted a similar analysis constraining geomorphic setting to rocky mountain streams and investigated the effect of restricting the range of data on model prediction accuracy. This restriction resulted in greatly improved accuracy when $Q > 0.1\ m^3/s$, thus demonstrating that limiting the range of Q and geomorphic condition can increase Q prediction accuracy. However, none of the above studies were limited to sand-bed streams. Properties of sand-bed streams, such as variable n_M at contrasting Q, or the necessity of quantifying bed particle size distribution, may have precluded model construction on these systems. Here, we describe predictive models developed for sand-bed streams in the Southeastern Plains, U.S. (SE Plains). This study is the first to develop empirical Q estimation models for sand-bed streams incorporating only A_c, R_H, and S without the need to estimate bed particle size distribution, or n_M. Our primary objective was to develop and validate empirical models for estimating Q from easily measured channel geometry and GIS variables to overcome known difficulties in estimating n_M in low-gradient, sand-bed streams of the

SE Plains ecoregion. The utility of such models will be to increase discharge estimation accuracy when empirical characterization of in-bank, high-flow events is unattainable or impractical.

METHODS

Data for model construction came from 75 sand-bed streams (i.e., streams with $d_{50} \leq 2$ mm or visually field verified; Figures 1A and 1B) comprising 484 Q measurements, which represented model training data. Sixty-nine of these sites were in the U.S. coastal plains including Florida (10 sites), Alabama (21), Georgia (12), South Carolina (15), and North Carolina (11) (Figure 2A). The coastal plains physiographic province consists of the SE Plains and Southern Coastal Plain ecoregions (Fenneman, 1917; Omernik, 1987). Florida sites were in the Southern Coastal Plain and all other sites were in the SE Plains. We included six sand-bed streams from New Zealand (Hicks and Mason, 1991) (Figure 2B).

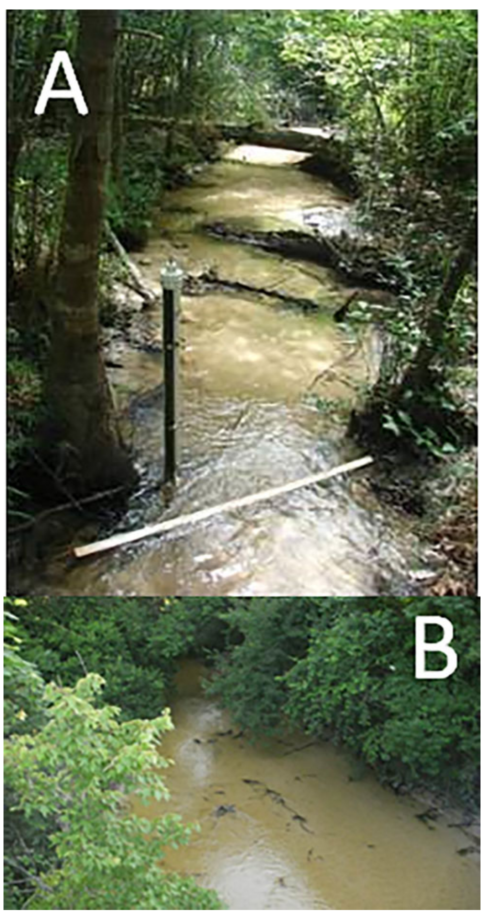

FIGURE 1. Small (A) and Large (B) Sand-Bed Streams in the SE Plains Typical of Our Study.

Published data from New Zealand (Hicks and Mason, 1991) and Florida (Gillen, 1996) were included to increase the upper range of Q values not provided by the SE Plains data, and thus broaden model applicability. Fifty-nine of the 69 sites in the U.S. coastal plains were unpublished data we collected from the SE Plains. Data from these 59 sites were from small watersheds (median area = 12.72 km^2, Strahler (1952) order 1-4), and were generally forested (personal observation). Model validation sites from the SE Plains, independent of the training data, consisted of 33 U.S. Geological Survey (USGS) sites, and an additional 22 sites we collected; the total number of Q values from validation sites was 317.

The training database consisted of three hierarchical groups of measurements: reduced, southeastern, and full database (Table 1). We used these groups to investigate if decreasing the range of modeled data resulted in higher prediction accuracy. The *reduced database* consisted of the 344 Q measurements from 59 unpublished sites we collected, with the largest observed Q = 2.64 m^3/s. The *southeastern database* included an additional 104 Q measurements from the 10 Florida sites reported in Gillen (1996), with the largest observed Q = 85.2 m^3/s. The *full database* included an additional 36 Q measurements from six New Zealand sites reported in Hicks and Mason (1991), with the largest observed Q = 874 m^3/s.

For data we collected in the SE Plains, Q was estimated at cross sections perpendicular to the direction of flow using the velocity-area method (Gore, 1996) at fixed intervals across the channel. Velocity (V) was quantified with a FlowMate current meter (Hach Company, Loveland, Colorado) and depth was measured to the nearest 0.5 cm. Wetted channel area was estimated by summing the area of the width × depth trapezoids formed by the water surface and bed, respectively, for each Q cell (A_{cell}). Q was estimated by summing $V \times A_{\text{cell}}$ for the cross section. We estimated the wetted perimeter (P_{w}) by summing the bed segments for each Q cell from water surface to water surface. Hydraulic radius (R_{H}) was calculated as $R_{\text{H}} = A_{\text{c}}/P_{\text{w}}$. R_{H}, Slope (S), A_{c}, and Q were reported in the New Zealand and Florida datasets. For methods used to quantify these variables, the reader is directed to Hicks and Mason (1991) and Gillen (1996).

We assumed that energy slope was equal to bed slope (S), with S estimated from high-resolution digital elevation models (usually 10 m) or from S values reported in the National Hydrography Plus Dataset (Horizon Systems Cooperation, *accessed* April 1, 2012, http://www.horizon-systems.com/nhdplus/index.php) in GRASS GIS (Neteler *et al.*, 2012). Map-derived S can be used to achieve similar Q prediction accuracy to slopes measured empirically (Bjerklie *et al.*, 2003, 2005).

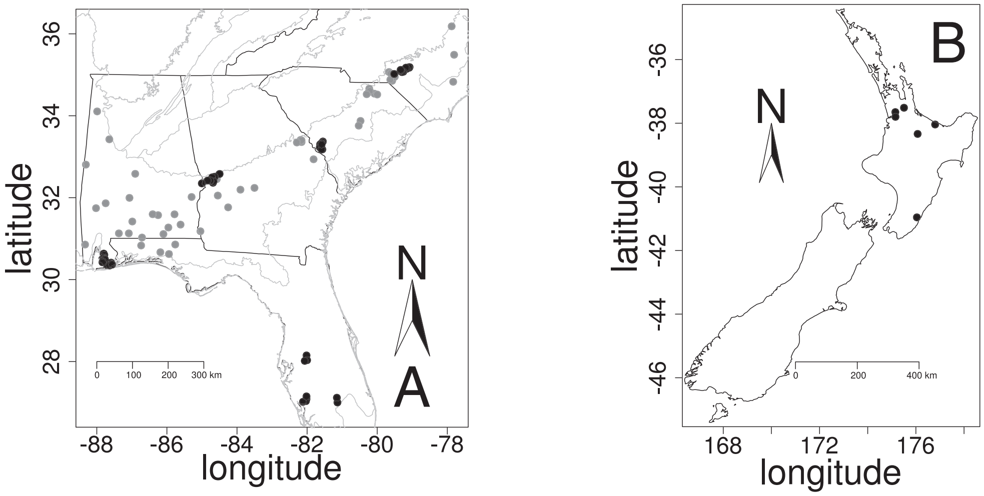

FIGURE 2. Training and Validation Sites Used in This Study. Southeastern, U.S. (A) and New Zealand (B) model fit sites (●) and model validation sites (●). U.S. Level III ecoregions are represented as lines in (A). All validation sites were in the Southeastern Plains, U.S., below the fall line and above the Southern Coastal Plain.

TABLE 1. Range of Hydraulic Variables in This Study.

Database	Q (m^3/s)	A_w (km^2)	W (m)	A_c (m^2)	R_H (m)	S (m/m)
Reduced	0.00018-2.64	0.59-43.92	0.81-17.70	0.029-10.01	0.030-0.65	0.00008-0.04055
Southeastern	0.00018-85.23	0.59-2139	0.81-50.60	0.029-121.7	0.030-2.17	0.00008-0.04055
Full	0.00018-874	0.59*	0.81*	0.029-855	0.030-5.25	0.00008-0.04055
Independent	0.00053-393.6	0.039-320.5	0.84-120.7	0.039-350.5	0.031-6.75	0.00002-0.01223

Notes: Q is the discharge, A_w is the watershed area, W is the wetted channel width, A_c is the wetted channel area, R_H is the channel hydraulic radius, and S is the slope.
*Upper range not reported in Hicks and Mason.

We constructed predictive models as follows. First, we used a simple logarithmic function based on exploratory graphical analysis of empirical Q data (Model 4). Second, we fit three models from Bjerklie *et al.* (2003, 2005; Models 5-6) and Dingman and Sharma (1997; Model 7) to data from the three databases after transforming them with the natural logarithm (Models 5-7). We fit models using ordinary least squares regression (OLS) (function lm; R Core Team, 2014) and assessed OLS assumptions with standard residual plots.

$$\ln Q = K + a\ln A_c \tag{4}$$

$$\ln Q = K + \ln A_c + b\ln R_H + c\ln S \tag{5}$$

$$\ln Q = K + a\ln A_c + b\ln R_H + c\ln S \tag{6}$$

$$\ln Q = K + a\ln A_c + b\ln R_H + c\ln^2 S \tag{7}$$

For each model investigated, we fit two models with K set to 0 to investigate model fit through the origin and K estimated with OLS to investigate model performance under these two contrasting conditions. We modified the leave-one-out jackknife method (McCuen, 2005) by leaving an entire site out at a time ("leave-one-site-out jackknifing") to assess model predictive accuracy. Briefly, we systematically excluded all observations from one of n sites (n = 75 for training data) from the database and fit the model to the remaining (n − 1) sites, a step we repeated until all sites had been removed once. Exclusion of entire sites ensured that the data used to estimate goodness-of-fit statistics for the model were independent from those used to develop the model. We then used variables from the excluded site to predict Q from the model, followed by calculating several diagnostic statistics to assess model predictive accuracy (Table 2). Goodness-of-fit statistics were calculated using the antilogs of simulated and observed Q values, Q_{sim} and Q_{obs}, respectively. We used Akaike's information criteria (AIC), calculated following Lopez *et al.* (2007) (Table 2), to compare models fit to the same dataset, whereas we used root mean square error (RMSE) to compare predictive accuracy of models fit on the same and different datasets (Table 2). We used AIC because

TABLE 2. Definitions of Model Fit Statistics and Fit Indication.

Model Fit Statistic	Abbreviation	Equation	Source	Better Fit
Nash-Suttcliffe Efficiency	NSeff	$1-[\sum(Q_{\mathrm{obs},i} - Q_{\mathrm{sim},i})^2]/[\sum(Q_{\mathrm{obs},i} - \mathrm{E}(Q_{\mathrm{obs}}))^2]$	Kalin *et al.* (2010)	Approaches 1
Mass balance error	MBe	$\sum(Q_{\mathrm{sim},i} - Q_{\mathrm{obs},i})/\sum Q_{\mathrm{obs},i}$	Kalin *et al.* (2010)	Approaches 0
Coefficient of determination	R^2	$[\sum((Q_{\mathrm{sim},i} - \mathrm{E}(Q_{\mathrm{obs}}))^2]/[\sum(Q_{\mathrm{obs},i} - \mathrm{E}(Q_{\mathrm{obs}}))^2]$	Kalin *et al.* (2010)	Approaches 1
Residual	e_i	$Q_{\mathrm{obs},i} - Q_{\mathrm{sim},i}$		Approaches 0
Mean residual	e_r	$\mathrm{E}(e)$	Dingman and Sharma (1997)	Approaches 0
Intercept of the $Q_{\mathrm{obs}} \sim Q_{\mathrm{sim}}$	$\mathrm{Int_{O\text{-}S}}$	β_0	Dingman and Sharma (1997)	Approaches 0
Slope of the $Q_{\mathrm{obs}} \sim Q_{\mathrm{sim}}$	$\mathrm{S_{O\text{-}S}}$	β_1	Dingman and Sharma (1997)	Approaches 1
Standard deviation residuals	SDE	$\mathrm{SD}(e)$	Dingman and Sharma (1997)	Approaches 0
Root mean square error	RMSE	$[\mathrm{E}^2(e) + \mathrm{SDE}^2]^{0.5}$	Dingman and Sharma (1997)	Approaches 0
Akaike's information criteria	AIC	$n^*\ln(\sum(Q_{\mathrm{obs},i} - Q_{\mathrm{sim},i})^2/n - k - 1) + 2 \times (k + 1)$	Lopez *et al.* (2007)	Decreases
Bayesian information criteria	BIC	$n^*\ln(\sum(Q_{\mathrm{obs},i} - Q_{\mathrm{sim},i})^2/n - k - 1) + k\ln(n)$	Lopez *et al.* (2007)	Decreases

Note: $\mathrm{E}(x)$ is the mean value of x, $\mathrm{SD}(x)$ is the standard deviation of x, k is the number of parameters fit in a given regression model, n is the number of observations. Indices have been omitted from summation operator for condensed presentation.

it selects the model explaining the most information in the empirical data out of the set of models investigated when the value is at a minimum (Burnham and Anderson, 2002). In addition, RMSE is useful for comparing models because it provides a measure of predictive uncertainty in the same units as the dependent variable. Both AIC and RMSE are designed to decrease as predictive accuracy increases (Helsel and Hirsch, 2002). Nonparametric bias correction was assessed to investigate the potential bias introduced by ln transformation and the effect on jackknife predictive accuracy using the method reported in Helsel and Hirsch (2002). The correction factor, i.e., $B = (\sum(Q_{\mathrm{obs}} - Q_{\mathrm{sim}}))/N$, is multiplied by Q_{sim} resulting in a corrected estimate. We used this procedure because Dingman and Sharma (1997) used it to correct their regression models; however, they did not assess whether this correction increased prediction accuracy.

In addition to the metrics summarized in Table 2, robust summaries of % error, i.e., $100 \times (Q_{\mathrm{sim}} - Q_{\mathrm{obs}})/Q_{\mathrm{obs}}$ also were used to assess model predictive accuracy. In doing this, median instead of the mean and Rousseeuw and Croux's (1993) Qn scale estimator (Qn) instead of the standard deviation were used as robust measures of location and scale because of the potential outliers in USGS data (Asquith *et al.*, 2013). We used median and Qn of % error for both the jackknifed training data and the independent data for comparability. Summarizing % error with previously described robust statistics instead of removing outliers was done because these statistics perform similar to the mean and standard deviation when the data do not contain outliers; however, when outliers exist, they perform better than their nonrobust equivalents (Maronna *et al.*, 2006).

We calculated summary statistics of % error for 16 discrete segments of the data (bins) from $e^{-9.1}$ to $e^{7.1}$ by increasing the exponent of e by 1 (e.g., $e^{-9.1}$ m^3/s; $e^{-8.1}$ m^3/s; …; $e^{5.1}$ m^3/s; $e^{6.1}$ m^3/s; $e^{7.1}$ m^3/s). We used binning to divide the continuous Q interval to facilitate calculation of descriptive statistics (i.e., median and Qn of % error). We chose the number of bins to contain the range of Q values in the full database. Last, it was important to identify the lower bound of Q prediction accuracy; therefore, we used logarithmic bins to provide increased resolution regarding error statistics in the lower range of Q examined.

We compared prediction accuracy of our models with those of other published studies (Golubtsov, 1969; Riggs, 1976; Williams, 1978; Bray, 1979; Jarrett, 1984; Meunier, 1989; Sauer, 1990; Dingman and Sharma, 1997; Bjerklie *et al.*, 2003, 2005; Lopez *et al.*, 2007). Only Manning's equation required estimation of n_{M}, which was estimated as 0.035 (Table 6.5, page 248 in Dingman, 2009), and used to simulate the typical application of Manning's equation in sand-bed streams. This value was within the range calculated using the method of Cowan (1956) as reported in Arcement and Schneider (1989; n_{M} range 0.028-0.1105), and was used as the baseline to compare values derived from Manning's equation to our models. Manning's equation using OLS to fit n_{M} as a model parameter was not investigated because previous studies documented the functional relationship of Q with A_{c}, R_{H}, and S was different from that of Manning's equation (Dingman and Sharma, 1997; Bjerklie *et al.*, 2003, 2005; Lopez *et al.*, 2007; others). After fitting Model (5) to the full database, we tested the R_{H} and S coefficients for significant differences from those of Manning's equation.

We selected model validation sites from USGS reference gauges reported in Falcone *et al.* (2010), and from our sample of independent gauged sites (i.e., field-verified sand-bed streams; Figures 1A and 1B).

Only USGS Q values with quality scores of "good" or "excellent," and from sites with predominately sand substrate were included in the validation database. The quality scores represent Q values ≤ 5% of actual field-measured Q (USGS, National Water Information System, *accessed* June 6, 2013, http://waterdata.usgs.gov/nwis/help?codes_help#rated). We accessed ratings table data from the USGS website (USGS, National Water Information System, *accessed* June 6, 2013, http://waterwatch.usgs.gov/accessed from the gauge website "field measurements" link). R_{H} values were not given as part of the USGS ratings tables. All three equations given in Models (5-7) require R_{H}, so we constructed a model relating A_{c}, wetted width (W), and mean depth (D as A_{c}/W) to R_{H} from all entries in the full database that included these variables. We fit the model to ln-transformed data to linearize and satisfy OLS assumptions. The exponential form of the resulting equation was:

$$R_{\mathrm{H}} = (A_{\mathrm{c}}^{0.24} D^{0.59})/1.79W^{0.0095};\ (R^2 = 0.97, n = 418). \quad (8)$$

Equation (8) was used to predict R_{H} from USGS empirical hydraulic data, as D was likely different from R_{H} in irregular channels. We then used these data (USGS and independent gauged sites, above) to compare predictive accuracy of the best model developed in this study with those of previous studies. Error properties were assessed by applying the equation developed in this investigation to new data and calculating goodness-of-fit metrics. All analyses were conducted in the R language for statistical computing (Ihaka and Gentleman, 1996; R Core Team, 2014).

RESULTS

Examination of residual plots indicated that OLS assumptions of independence and normality of residuals were met for models developed in this paper, although plots suggested some heteroscedasticity (Figures 3A-3F). With heteroscedasticity present, OLS still provided unbiased parameter estimates (Montgomery *et al.*, 2006).

Prediction accuracy increased with K as a free parameter when Model (4) was fit to all databases,

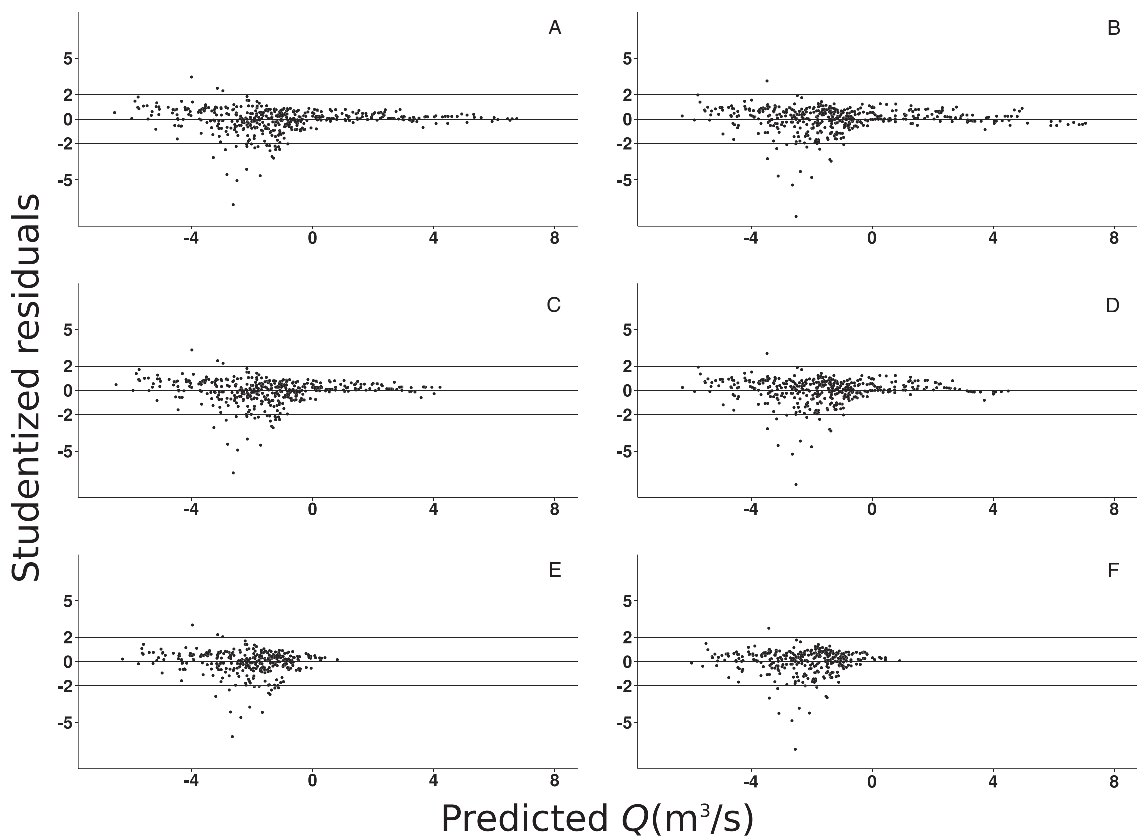

FIGURE 3. Predicted ln$Q$ *vs*. Studentized Residuals for Models Developed in This Study. (A) and (B) are Model (5) and (4) fit to the full database, (C) and (D) are Model (5) and (4) fit to the southeastern database, and (E) and (F) are Model (5) and (4) fit to the reduced database, respectively.

TABLE 3. Results for Jackknifed Model Fits.

	Reduced Database		Southeastern Database		Full Database	
Equation	**(4)**	(5)	**(4)**	(5)	**(5)**	(4)
NSeff	0.75	0.70	0.95	0.92	0.99	0.80
MBe	−0.24	−0.17	−0.04	−0.17	−0.06	0.20
R^2	0.83	0.75	0.96	0.94	0.99	0.96
e_r	−0.06	−0.04	−0.09	−0.40	−1.06	3.19
$\mathrm{Int}_{\text{O-S}}$	0.04	0.06	−0.28	0.05	−0.57	−2.59
$\mathrm{S}_{\text{O-S}}$	0.62	0.60	1.08	0.81	0.97	1.35
SDE	0.17	0.19	1.85	2.30	8.99	35.31
RMSE (m^3/s)	0.18	0.20	1.85	2.33	9.05	35.45
AIC	−1,092	−1,042	524	715	2,007	3,247
BIC	−1,087	−1,037	530	721	2,014	3,254
N	325	325	418	418	454	454

Note: Bolded equation numbers represent models of highest predictive ability based on Akaike's information criteria (AIC) and smallest root mean square error (RMSE). N is the total number of observations. Definition of model fit statistics as in Table 2. Reduced, Southeastern, and Full databases are those reported in the text (also, see Table 1).

compared with K fixed at 0 (RMSE mean decrease = 24.77 m^3/s, AIC mean decrease = 1,047.49). In contrast, when K was set to 0 and Model (5) was fit to all databases, prediction accuracy increased compared with K estimated as a free parameter (RMSE mean decrease = 3.87 m^3/s, AIC mean decrease = 337.76). For these reasons, K was allowed to vary when fitting Model (4) and set to 0 when fitting Model (5) for the remainder of the study.

Of the models investigated and when fit to the full database, Model (5), $Q = A_c R_H^{0.6906} S^{0.1216}$, was the best model having the lowest RMSE and AIC. As a result of this, Model (5) was fit to the database under investigation for comparison with Model (4), which best fit the southeastern and reduced databases (Table 3). The bias corrected form of Model (5), fit to the full database, had lower predictive accuracy than the uncorrected version (uncorrected RMSE = 9.1 m^3/s *vs.* 23.2 m^3/s after bias correction). The reduced and southeastern databases were best modeled with a simple logarithmic function of A_c (Model 4). However, when fits were compared between Models (4) and (5) fit to the reduced and southeastern database, respectively, only a negligible increase in RMSE occurred: 0.02 and 0.48 m^3/s. In contrast, Model (5), fit to the full database, resulted in a decrease in RMSE of 26.4 m^3/s compared to the Model (4) fit (Table 3). The R_H coefficient of Model (5), fit to the full database, was not significantly different from the Manning's equation coefficient of 2/3 (p = 0.504). In contrast, the estimate of ~1/7 for the S coefficient was significantly different from the Manning's equation coefficient of 1/2 ($p < 2.2e^{-16}$).

When comparing fits of Model (5), there was strong concordance between Q_{sim} and Q_{obs} across all three databases (Figures 4A-4C). A Q value of 0.045 m^3/s should be considered at the lower end of acceptable model application, as % error did not stabilize until the 0.045 to 0.122 m^3/s bin (Figures 4D-4F). The lower end estimate of model applicability was concordant with the point of significant deviation from the 1:1 line (Figures 4A-4C). All three models showed an absolute value of median % error <24% in the 0.045 to 0.122 m^3/s bin (range = −13 to 24%), and all models had an absolute value of median % error <45% for all other bins with Q > 0.122 m^3/s. Model (5), when fit to the full database, consistently underestimated Q by <30% between 0.33 and 18.17 m^3/s (Figure 4D). At Q > 18.17 m^3/s, there also was underestimation bias, although it was consistently <20% (Figure 4D).

When Model (5), fit to the full database, and models from the literature were applied to independent data during model validation, Model (5) showed the highest predictive accuracy (i.e., lowest AIC and RMSE), and a Q_{obs}~Q_{sim} slope closest to 1 (Table 4). Furthermore, hydrologic indicators of goodness of fit, the Nash-Suttcliffe efficiency (0.94) and Mass balance error (0.14) were close to 1 and 0, respectively (Table 4). The Q_{obs}~Q_{sim} relationship using the independent data showed strong concordance (R^2 = 0.97, Figure 5A, Table 4). The absolute value of median error was below the jackknifed results ~62% of the time in the range of Q = 0.00224 to 874 m^3/s (compare Figures 4D and 5B). The absolute value of % errors for the validation dataset was <26% for Q = 0.00224 to 874 m^3/s, and resulted in reduced % error when compared to the jackknifed training data results. Percent error for the 0.000304 to 0.000825 m^3/s Q bin was the highest for all bins (median = 723%). The median % error for low Q (0.00224 to 1 m^3/s) was −16.45%, medium Q (1 to 50 m^3/s) was −19.68%, and high Q (>50 m^3/s) was 19.78%. This pattern suggests underestimation bias in the low and medium range, and an overestimation

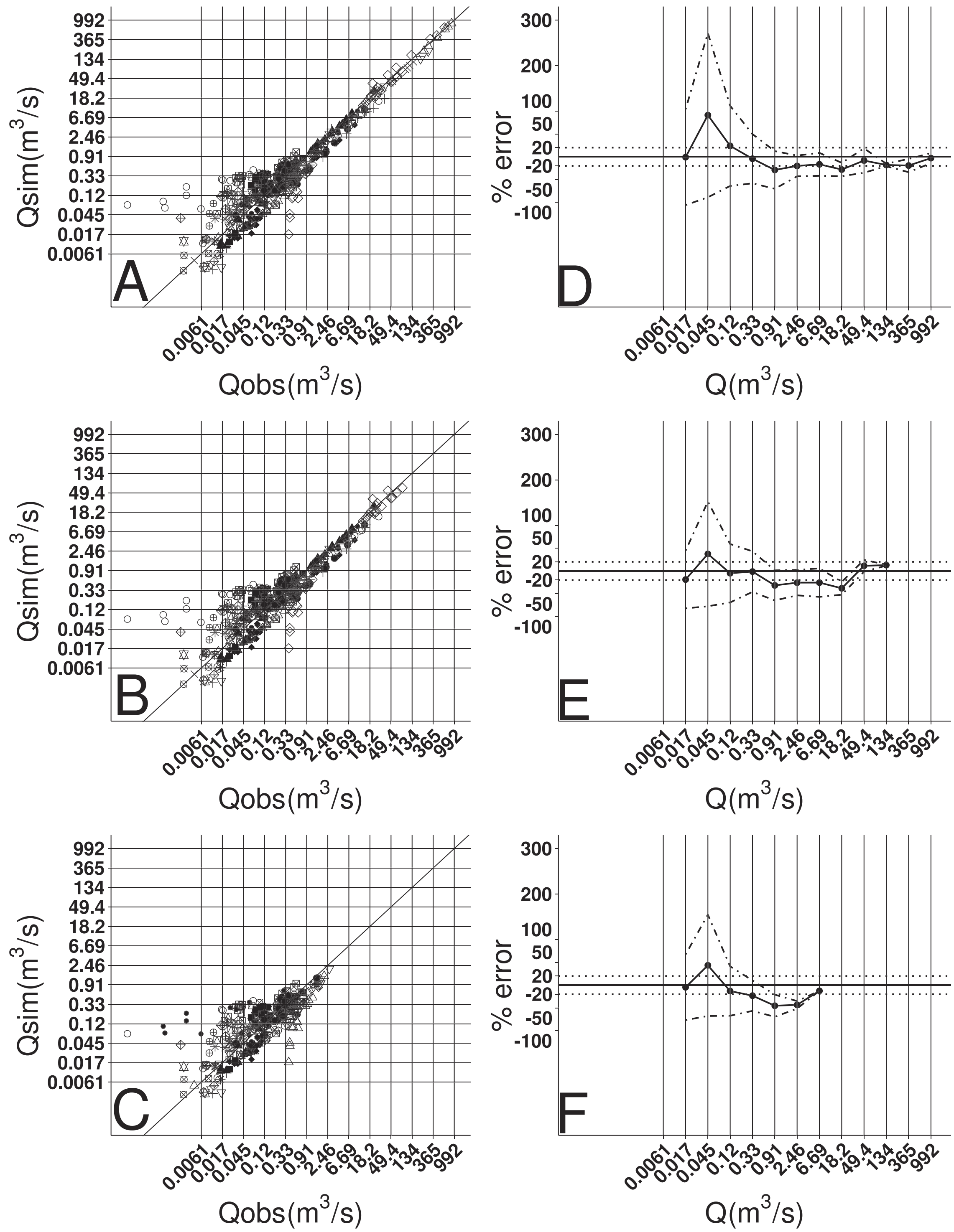

FIGURE 4. Model (5) Fit Results for All Three Databases Evaluated in This Study (A, full database; B, southeastern database; C, reduced database, see text) and Their Error Characteristics (D, full database; E, southeastern database; F, reduced database). (A-C) ln-Transformed Q_{obs} m^3/s *vs*. Q_{sim} m^3/s plots. Solid diagonal lines in A-C are the 1:1 lines. Solid vertical and horizontal lines in A-C and solid vertical lines in D-F represent percent error bins. Different symbols represent observations from different streams. (D-F) % Error plots where x-axis is Q m^3/s and y-axis is % error. Median % error (●) ± Qn (–·–) are plotted at the maximum of the bin in which they were calculated. (D-F) Median % error was >100% for Q bins < 0.0061 m^3/s and are not presented in the graph.

bias in the high range, but was within ±20% of Q_{obs}. Model (5) accurately predicted the independent data for $Q \geq 0.00224$ m^3/s during model validation (Figures 5A and 5B), showing higher prediction accuracy with the independent data than that suggested by jackknifing with test data.

TABLE 4. Validation Data Comparison of Model (5), Fit to the Full Database (see text), with Other Published Models.

Reference	Equation	NSeff	MBe	R^2	ε_r	$\mathrm{Int_{O\text{-}S}}$	$\mathrm{S_{O\text{-}S}}$	SDE	RMSE (m^3/s)	AIC	BIC	N
This study	$Q = A_c R_H^{0.6906} S^{0.1216}$	0.94	0.14	0.97	0.89	−0.04	1.14	6.68	6.74	1,217	1,222	317
Lopez *et al.* (2007)	$Q = 2.93 A_c^{1.02} R_H^{0.79} S^{-0.057 \ln S}$	0.65	−0.61	0.93	−3.97	−0.30	0.44	15.96	16.44	1,789	1,802	317
Bjerklie *et al.* (2005)	$Q = 4.84 A_c^{1.10} R_H^{0.53} S^{0.33}$	0.63	0.50	0.97	3.29	−0.28	1.54	16.65	16.97	1,809	1,822	317
Williams (1978)*	$Q = 4.0 A_c^{1.21} S^{0.28}$	0.58	0.98	0.93	6.40	3.39	1.46	16.85	18.03	1,844	1,853	317
Bjerklie *et al.* (2005)	$Q = 7.14 A_c R_H^{0.67} S^{0.33}$	0.50	0.62	0.97	4.09	−0.10	1.64	19.26	19.69	1,900	1,909	317
Bjerklie *et al.* (2003)	$Q = 7.22 A_c^{1.02} R_H^{0.72} S^{0.35}$	0.37	0.60	0.98	3.94	−0.86	1.73	21.69	22.05	1,975	1,988	317
Dingman and Sharma (1997)	$Q = 1.56 A_c^{1.17} R_H^{0.40} S^{-0.0543 \ln S}$	0.23	−0.82	0.87	−5.36	0.24	0.15	23.80	24.39	2,039	2,052	317
Sauer (1990)*	$Q = 8.33 A_c R_H^{0.59} S^{0.32}$	0.20	0.92	0.97	6.05	0.75	1.81	24.13	24.87	2,048	2,057	317
Riggs (1976)*	$Q = 1.55 A_c^{1.33} S^{0.05-0.056 \ln S}$	−0.06	−1.00	0.00	−6.56	0.00	0.00	27.81	28.57	2,139	2,152	317
Manning (1891)	$Q = (1/n_M) A_c R_H^{2/3} S^{1/2}$	−0.51	1.11	0.96	7.30	−0.07	2.12	33.39	34.18	2,247	2,252	317
Bray (1979)*	$Q = 6.17 A_c R_H^{1/2} S^{0.24}$	−0.50	1.45	0.97	9.53	2.26	2.11	32.73	34.09	2,248	2,257	317
Bray (1979)*	$Q = 7.96 A_c R_H^{0.60} S^{0.29}$	−0.68	1.32	0.97	8.67	0.80	2.20	34.96	36.02	2,283	2,292	317
Bray (1979)*	$Q = 9.62 A_c R_H^{2/3} S^{0.32}$	−1.12	1.35	0.97	8.87	−0.09	2.37	39.51	40.49	2,357	2,366	317
Lopez *et al.* (2007)	$Q = 5.56 A_c^{1.03} R_H^{0.77} S^{0.27}$	−2.53	1.46	0.98	9.58	−2.21	2.80	51.39	52.28	2,522	2,535	317
Lopez *et al.* (2007)	$Q = 6.04 A_c R_H^{0.82} S^{0.26}$	−3.42	1.65	0.98	10.79	−2.40	3.01	57.44	58.44	2,590	2,599	317
Golubtsov (1969)*	$Q = 4.50 A_c R_H^{2/3} S^{1/6}$	−6.02	2.54	0.97	16.67	0.18	3.52	71.74	73.65	2,736	2,746	317
Jarrett (1984)*	$Q = 3.17 A_c R_H^{0.83} S^{0.12}$	−13.35	3.09	0.97	20.27	−3.50	4.63	103.34	105.31	2,963	2,972	317
Meunier (1989)*	$Q = 1.3 A_c R_H^{0.86} S^{-0.084}$	−77.05	7.46	0.95	48.90	−5.94	9.37	240.73	245.65	3,500	3,509	317

Note: All models are in SI units (*references indicate models that were reported in Lopez *et al.* (2007) in SI units). N is the total number of observations. Definition of model fit statistics as in Table 2.

DISCUSSION AND CONCLUSIONS

The hydraulic models developed in our study should be of great utility in estimating at-a-site point discharges when compared to regional regression models or surface water hydraulic models. Regional regression models are simple to apply, but, generally, are used to estimate gross flow parameters such as flood recurrence (Riggs, 1973) or annual mean flow (Vogel *et al.*, 1999) from basin and climatic variables. Surface water hydraulic models, such as HEC-RAS (USACE, 2010), require estimation of n_M. The need to estimate n_M limits its utility because of the error associated with highly mobile sand beds.

The jackknifed and independent data model goodness-of-fit statistics indicated that Model (5), when fit to the full database, had high prediction accuracy compared with all other models investigated. This result suggests that this model can be used reliably to predict in-bank discharge from hydraulic characteristics of sand-bed streams in the SE Plains ecoregion. Model (5), fit to the full database, should be preferred for application within the SE Plains ecoregion over all previously published models as it showed the highest predictive accuracy with completely independent data from sites across the expanse of this region. Percent error for training data was <29% when $0.045\ m^3/s < Q < 874\ m^3/s$. There was no reason to favor models from the reduced databases, as % error patterns were similar among the three databases. RMSE was higher for the larger (full) database, but this result could have been produced by a larger Q range (Table 1). The validation data showed that $Q \geq 0.00224\ m^3/s$ showed an absolute value of % error <26%, suggesting that Model (5), fit to the full database, showed good predictive accuracy. Furthermore, validation data resulted in % error within 20% for low, medium, and high Q ranges demonstrating the utility of Model (5), fit to the full database, for predicting a wide range of Q. Furthermore, Sauer and Meyer (1992) reported that errors in empirically measured Q can range from 2 to 20% with a typical value of 2-3%. In addition, Pelletier (1988) reported a similar range with a median value of ~6%. The model presented in this study overlaps with high end error estimates for empirically determined Q and, thus should be highly applicable to estimating Q in SE Plains streams when logistics or safety do not allow for in-bank measurement.

The equation that showed the highest predictive accuracy given RMSE, AIC, and other goodness-of-fit indicators was based on Model (5) from Bjerklie *et al.* (2003). The resulting equation corresponding to the validated Model (5), fit to the full database, is as follows:

$$Q = A_c R_H^{0.6906} S^{0.1216}. \tag{9}$$

The above-derived equation should be used for in-bank discharge prediction in unbraided, low-gradient, sand-bed streams of the SE Plains. Conservatively, we suggest that this equation only be used to predict discharge >0.045 m^3/s even though independent data

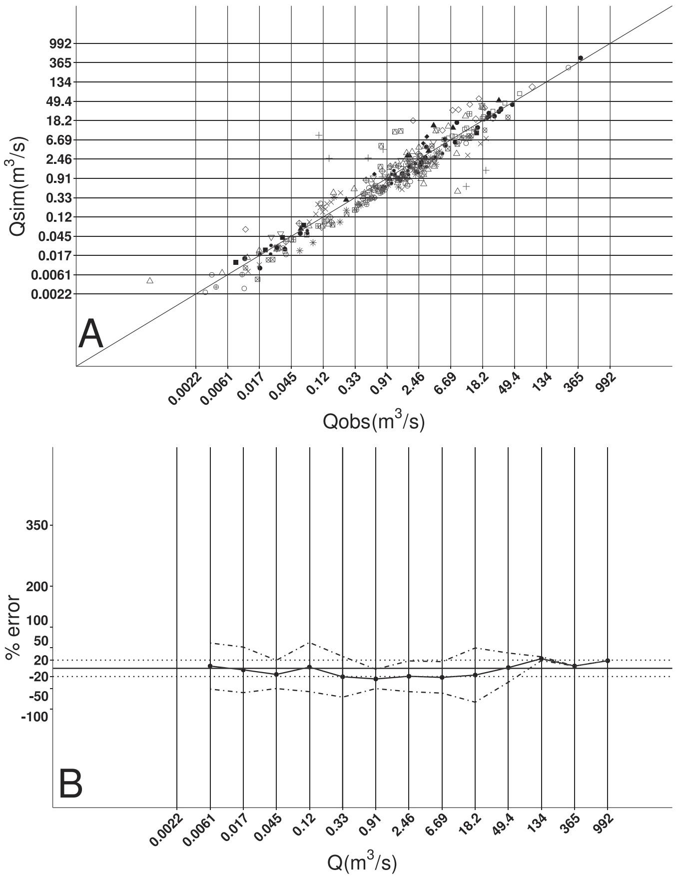

FIGURE 5. Results for Model (5) Applied to Validation Data (A) and Error Characteristics (B). (A) ln-Transformed Q_{obs} m^3/s^1 *vs.* Q_{sim} m^3/s plots. Solid diagonal line in A is the 1:1 line. Solid vertical and horizontal lines in A and solid vertical lines in B represent percent error bins. Different symbols represent observations from different streams. (B) % Error plots where x-axis is Q m^3/s and y-axis is % error. Median % error (●) ± Qn (–·–) plotted at the maximum of the bin where they were calculated. (B) Median % error was >100% for Q bins <0.0022 m^3/s and are not presented in figure.

showed high prediction accuracy >0.00224 m^3/s. We suggest this higher value because both the jackknifed and validation % errors stabilized by this point and should result in accurate Q estimates, and validation errors were within ±20%. Furthermore, Equation (9) showed a higher predictive accuracy and a different empirical relationship for S than the Manning equation over the full range of Q in the sand-bed systems we examined. Even though RMSE increased with increasing range of modeled Q, Equation (9) should be useful for Q prediction in sand-bed streams because of increased applicability and error characteristics comparable or better than the reduced range models. Constraining the database geomorphically to sand-bed streams resulted in higher predictive accuracy of models fit in this study compared to models from unconstrained geomorphic settings.

A major advantage of using Equation (9) over typical Q-estimating methods is that it (1) does not require estimation of n_{M}, thus removing a major source of error, (2) does not require quantification of bed particle size distribution as in the Brownlie (1983) equation, thus making it useful for regional assessments involving quantifying Q on large geographic scales, (3) incorporates hydraulic variables (area, wetted perimeter, and slope) that are easily measured in the field and/or developed from GIS-derived layers (e.g., S values), and (4) shows the highest predictive accuracy of all similar models compared from the published literature for Q estimation. In particular, Equation (9) should be highly appropriate for, and effective in, water resource management plans involving estimating Q in sand-bed streams of the SE Plains. Furthermore, this equation has been validated with independent data spanning a large geographic range (Alabama, Florida, Georgia, South Carolina, and North Carolina) and provides error estimates for a range of Q. This equation should increase accuracy of stream gauging and also aid in describing flow regime-ecology linkages; as a result, we recommend its use, testing, and continued refinement in these and other low-gradient sand-bed systems.

ACKNOWLEDGMENTS

We thank Andrew Morrison, S. Lawrence Dingman, and David Bjerklie for providing hydraulic data. We also thank three anonymous reviewers whose comments were valuable in improving earlier versions of this manuscript. This research was funded by SERDP grant SI-1694.

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